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Analytic Description of the Motion of a Trapped Ion in an Even or Odd Coherent State

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ABSTRACT

A completely analytic description is given of the motion of a trapped ion which is in either an even or an odd coherent state. Comparison to recent theoretical and experimental work is made.

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Recently, Matos Filho and Vogel [1] gave an analysis of a trapped ion [2] which, to very high precision, is in an even (+) or odd (-) coherent state [3]:

$$\psi_+ \ = [\cosh|\alpha|^2]^{-1/2}\sum_{n=0}^{\infty}\frac{\alpha^{2n}}{\sqrt{(2n)!}}|2n\rangle \ , \tag{1}$$

$$\psi_- \ = [\sinh|\alpha|^2]^{-1/2}\sum_{n=0}^{\infty}\frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}}|2n+1\rangle \ . \tag{2}$$

Specifically, they gave lovely three-dimensional numerical graphs of the probability densities and Wigner functions, for the even and odd cases, as functions of position and time, for particular values of $\alpha = \alpha_1 + i\alpha_2$.

Here it is noted that closed-form expressions can be given for these wave functions, and hence for the probability densities and the Wigner functions. A direct way to evaluate the above sums [4] is to use generating function techniques [5], yielding

$$\psi_+ = \left[\frac{e^{-\alpha^2}}{\pi^{1/2}\cosh|\alpha|^2}\right]^{1/2} e^{-x^2/2}\cosh(\sqrt{2}\alpha x) \ , \tag{3}$$

$$\psi_- = \left[\frac{e^{-\alpha^2}}{\pi^{1/2}\sinh|\alpha|^2}\right]^{1/2} e^{-x^2/2}\sinh(\sqrt{2}\alpha x) \ . \tag{4}$$

(One could, of course, obtain these expressions by other methods [3, 6, 7].) Physical intuition is satisfied when the above expressions are transformed to two Gaussians displaced on opposite sides of the origin [7]. Ignoring $e^{-i4\alpha_1\alpha_2}$,

$$\psi_\pm = \left[2\pi^{1/2}(1 \pm e^{-2|\alpha|^2})\right]^{-1/2}\left[e^{-(x-\sqrt{2}\alpha_1)^2/2+i\sqrt{2}\alpha_2 x} \pm e^{-(x+\sqrt{2}\alpha_1)^2/2-i\sqrt{2}\alpha_2 x}\right] \ . \tag{5}$$

Time-dependence can be included by letting $\alpha \to \alpha\exp[-i\omega t]$. We now take the convention $\alpha \to \alpha_0$ is real, as was done in Ref. [1]. Then it is straighforward to calculate additional quantities. For example, the probability densities are

$$\rho_+ = \frac{e^{\alpha_0^2[\sin^2\omega t - \cos^2\omega t]}}{\pi^{1/2}[e^{\alpha_0^2}+e^{-\alpha_0^2}]}e^{-x^2}[\cosh\{2\sqrt{2}\alpha_0(\cos\omega t)x\} + \cos\{2\sqrt{2}\alpha_0(\sin\omega t)x\}], \tag{6}$$

$$\rho_- = \frac{e^{\alpha_0^2[\sin^2\omega t - \cos^2\omega t]}}{\pi^{1/2}[e^{\alpha_0^2} - e^{-\alpha_0^2}]} e^{-x^2}[\cosh\{2\sqrt{2}\alpha_0(\cos\omega t)x\} - \cos\{2\sqrt{2}\alpha_0(\sin\omega t)x\}]. \qquad (7)$$

The above functions ρ_+ and ρ_+ describe the forms of Figs. 1 and 4 in Ref. [1]. We show them in our Figs 1 and 2. Note in particular that the terms $\exp[-x^2] \times$ cosh describe the two "wave-packets" on opposite sides of the origin. The cos terms describe the interference effects near $x = 0$ at $t = (2j + 1)\pi/2$.

These even and odd coherent states have recently been created by Wineland's group [2]. They took a trapped ${}^9Be^+$ ion and first cooled it to its zero-point energy. Then they created the even/odd states by a series of laser pulses that entangled the electronic and motional states of the ion. The separate packets were separated by as much as 800 Åwith their individual sizes at 70 Å.

This same group has produced squeezed ground states [8]. Therefore, it is possible that squeezed even and odd states [9] may be produced in the future.

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Figure Captions

Figure 1. A three-dimensional plot of the even-coherent-state probability density, ρ_+, as a function of position, x, and time, t, for $\alpha_0 = 2$.

Figure 2. A three-dimensional plot of the odd-coherent-state probability density, ρ_-, as a function of position, x, and time, t, for $\alpha_0 = 5^{1/2}$.

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